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21MAT21

Second Semester B.E. Degree Examination, July/August 2022 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dx dy dz$. (06 Marks)
- b. Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy dy dx$ by changing the order of integration. (07 Marks)
- c. Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. (07 Marks)

OR

- 2 a. Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$ by changing to polar coordinates. (06 Marks)
- b. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (07 Marks)
- c. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\cot \theta} d\theta = \frac{\pi}{\sqrt{2}}$. (07 Marks)

Module-2

- 3 a. Find the directional derivative of $\phi = \frac{xz}{x^2 + y^2}$ at the point (1, -1, 1) in the direction of $\hat{i} - 2\hat{j} + \hat{k}$. (06 Marks)
- b. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \text{grad}(xy^3z^3)$. (07 Marks)
- c. If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that \vec{F} is irrotational. (07 Marks)

OR

- 4 a. If $\vec{F} = xy\hat{i} + (x^2 + y^2)\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ along the curve $C: y = x^2 - 4$ in the xy-plane from the point (2, 0) to (4, 12). (06 Marks)
- b. Using Green's theorem, evaluate $\int (y - \sin x) dx + \cos x dy$ where C is the triangle in the xy-plane bounded by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. (07 Marks)
- c. Using Stokes theorem, evaluate $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken around the rectangle bounded by $x = 0$, $x = a$, $y = 0$, $y = b$. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-3

- 5 a. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$ (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that $x = 0, z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$. (07 Marks)
- c. Derive one dimensional wave equation, $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. (07 Marks)

OR

- 6 a. Form the partial differential equation by eliminating the arbitrary function from, $x+y+z = f(x^2+y^2+z^2)$ (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$ when y is an odd multiple of $\frac{\pi}{2}$. (07 Marks)
- c. Solve $(x + 2z)p + (4zx - y)q = (2x^2 + y)$ (07 Marks)

Module-4

- 7 a. Find a root of the equation $\tan x = x$ which is near to $x = 4.5$ using Newton's Raphson method. (06 Marks)
- b. Given $\sin 45^\circ = 0.7071, \sin 50^\circ = 0.7660, \sin 55^\circ = 0.8192, \sin 60^\circ = 0.8660$ find $\sin 52^\circ$ using Newton's forward interpolation formula. (07 Marks)
- c. Evaluate $\int_0^1 \sqrt{\sin x + \cos x} dx$ correct to two decimal places using Simpson's $\frac{1}{3}$ rule taking seven Equi distance ordinates. (07 Marks)

OR

- 8 a. Find the root of the equation $x \log_{10} x = 1.2$ that lies between 2 and 3 correct to three decimal places, using Regula Falsi method. (06 Marks)
- b. Using Newton's divided difference formula find $f(4)$ given that,

x	0	2	3	6
f(x)	-4	2	14	158

- c. Evaluate $\int_0^{0.3} \sqrt{1-8x^3} dx$ using Simpson's $\left(\frac{3}{8}\right)^{\text{th}}$ rule by taking seven ordinates. (07 Marks)

Module-5

- 9 a. Solve $\frac{dy}{dx} = e^x - y, y(0) = 2$ using Taylor's series method upto 4th degree terms at any point x . (06 Marks)
- b. Using modified Euler's method, find y at $x = 0.2$ from $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0) = 1$ taking $h = 0.1$ perform two iteration at each step. (07 Marks)
- c. Solve $\frac{dy}{dx} = 2e^x - y$ given that $y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090$ find $y(0.4)$ using Milne's predictor corrector method. (07 Marks)

OR

- 10 a. Employ Taylor's series method to obtain the value of y at $x = 0.1$ for the equation $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$ considering upto 4th degree term. (06 Marks)
- b. Use Runge Kutta method of order 4 find y at $x = 0.2$ given that $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$ taking $h = 0.2$. (07 Marks)
- c. Apply Milne's predictor corrector method to find $y(1.4)$ from $\frac{dy}{dx} = x^2 + \frac{y}{2}$ given that $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4549$, $y(1.3) = 2.7514$. (07 Marks)
